

2290

logarithms

Go H.O. 2021

4

$$3) \log_{12} 2 + \log_{12} 42 = \log_{12} (2 \cdot 42) = \log_{12} 84 = \log_{12} 12^2 = 2 \log_{12} 12 = 2$$

$$4) \log_3 6 + \log_3 \frac{2}{3} = \log_3 \left( \frac{6}{3} \cdot \frac{2}{3} \right) = \log_3 2 = 2 \log_3 \sqrt{2} = 2$$

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$$3) \log_{\frac{1}{3}} 54 - \log_{\frac{1}{3}} 2 = \log_{\frac{1}{3}} (54 : 2) = \log_{\frac{1}{3}} 27 = \log_{\frac{1}{3}} \left( \frac{1}{3} \right)^{-3} = 3$$

$$4) \log_8 \frac{1}{16} - \log_8 32 = \log_8 \left( \frac{1}{16} : 32 \right) = \log_8 \left( \frac{1}{16 \cdot 32} \right) = \log_8 8^{-3} = -3 \log_8 8 = -3$$

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$$3) \log_{\frac{1}{3}} \sqrt[4]{27} = \log_{\frac{1}{3}} 3^{\frac{3}{4}} = \log_{\frac{1}{3}} \left( \frac{1}{3} \right)^{-\frac{3}{4}} = -\frac{3}{4} = -0.75$$

$$4) \log_2 \frac{1}{\sqrt[5]{128}} = \log_2 \frac{1}{2^{\frac{5}{4}}} = \log_2 2^{-\frac{4}{5}} = -\frac{4}{5} = -0.8$$

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$$3) \frac{1}{2} \log_7 36 = \log_7 14 - 3 \log_7 \sqrt[3]{21} = \log_7 36^{\frac{1}{2}} - \log_7 14 - \log_7 (\sqrt[3]{21})^3 =$$

$$= \log_7 6 - \log_7 14 - \log_7 21 = \log_7 (6 : 14 : 21) = \log_7 \left( \frac{3}{7} : 21 \right) = \log_7 \left( \frac{3}{7} \cdot \frac{1}{21} \right) = \log_7 \left( \frac{1}{49} \right) =$$

$$= \log_7 7^{-2} = -2$$

$$4) 2 \log_{\frac{1}{3}} 6 - \frac{1}{2} \log_{\frac{1}{3}} 400 + 3 \log_{\frac{1}{3}} \sqrt[3]{45} = \log_{\frac{1}{3}} 6^2 - \log_{\frac{1}{3}} 400^{\frac{1}{2}} + \log_{\frac{1}{3}} (\sqrt[3]{45})^3 =$$

$$= \log_{\frac{1}{3}} 36 - \log_{\frac{1}{3}} 20 + \log_{\frac{1}{3}} 45 = \log_{\frac{1}{3}} (36 : 20 \cdot 45) = \log_{\frac{1}{3}} 81 = \log_{\frac{1}{3}} \left( \frac{1}{3} \right)^{-4} = -4$$

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$$3) \frac{\log_5 36 - \log_5 12}{\log_5 9} = \frac{\log_5 \left( \frac{36}{12} \right)}{\log_5 \left( \frac{9}{3} \right)} = \frac{\log_5 3}{2 \log_5 3} = \frac{1}{2}$$

$$4) \frac{\log_7 8}{\log_7 15 - \log_7 30} = \frac{\log_7 8}{\log_7 \frac{15}{30}} = \frac{\log_7 2^3}{\log_7 2^{-1}} = \frac{3 \log_7 2}{-1 \log_7 2} = -3$$

2295

$$1) \log_a x = ? \quad \log_a b = 3, \log_a c = -2$$

$$x = a^3 b^2 \sqrt[3]{c} \rightarrow \log_a (a^3 b^2 \sqrt[3]{c}) = \log_a a^3 + \log_a b^2 + \log_a \sqrt[3]{c} =$$

$$= 3 \log_a a + 2 \log_a b + \frac{1}{3} \log_a c = 3 + 2 \cdot 3 + \frac{1}{3} (-2) = 3 + 6 - \frac{2}{3} = 9 - \frac{2}{3} = 8 \frac{1}{3}$$

$$2) x = \frac{a^4 \sqrt[3]{b}}{c^3} \rightarrow \log_a \left( \frac{a^4 \sqrt[3]{b}}{c^3} \right) = \log_a a^4 + \log_a \sqrt[3]{b} - \log_a c^3 =$$

$$= 4 \log_a a + \frac{1}{3} \log_a b - 3 \log_a c = 4 + \frac{1}{3} \cdot 3 - 3(-2) = 4 + 1 + 6 = 11$$